Optimization

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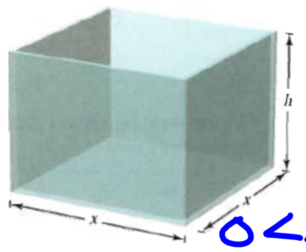


Open box with square base:
$S = x^2 + 4xh = 108$

A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

$0 < x < \sqrt{108}$

$V = x^2 h$ ① Primary equation

$108 = x^2 + 4xh$ ② Secondary eq.

$\frac{108 - x^2}{4x} = h$ ③ Solve 2nd eq. for a variable

$V = x^2 h$

$V = x^{\cancel{2}} \left(\frac{108 - x^2}{4\cancel{x}} \right)$ ④ plug into the primary & Simplify

$V = \frac{108x - x^3}{4}$

$V = 27x - \frac{1}{4}x^3$

$\frac{dV}{dx} = 27 - \frac{3}{4}x^2$ ⑤ derivative

$27 - \frac{3}{4}x^2 = 0$ ⑥ Critical Numbers

$27 = \frac{3}{4}x^2$

$\sqrt{36} = \sqrt{x^2}$

$\pm 6 = x$

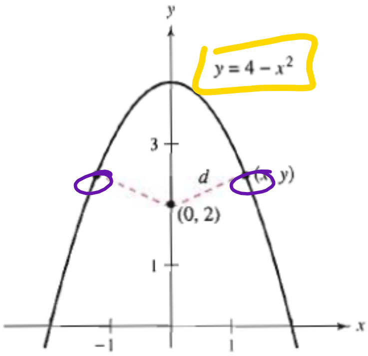
$\boxed{x = 6}$

$+ \quad 6 \quad -$ ⑦ 1st der test

$108 = x^2 + 4xh$ ⑧ find the other variables

$108 = 6^2 + 4(6)h$

$\boxed{h = 3}$



The quantity to be minimized is distance:
$d = \sqrt{(x-0)^2 + (y-2)^2}$.

Which points of the graph of $y=4-x^2$ are closest to the point (0,2)?

$d = \sqrt{(x-0)^2 + (y-2)^2}$ Minimize

$y = 4 - x^2$ (Secondary)

$d = \sqrt{x^2 + (4-x^2-2)^2}$ $2-x^2$

$d = \sqrt{x^2 + 4 - 4x^2 + x^4}$

$d = \sqrt{x^4 - 3x^2 + 4}$

$d' = (x^4 - 3x^2 + 4)^{\frac{1}{2}}$

$\frac{1}{2}(x^4 - 3x^2 + 4)^{-\frac{1}{2}}(4x^3 - 6x)$

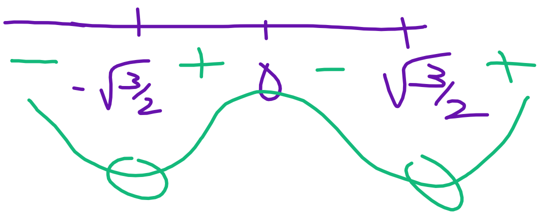
$d' = \dfrac{4x^3 - 6x}{2\sqrt{x^4 - 3x^2 + 4}} = 0$

$4x^3 - 6x = 0$

$2x(2x^2 - 3) = 0$

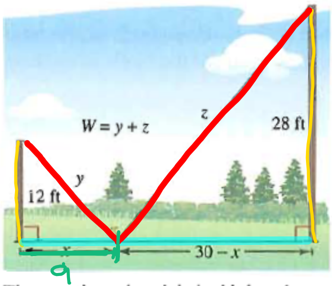
$x = 0 \quad x = \pm\sqrt{3/2}$

$x = 0,\ \sqrt{3/2},\ -\sqrt{3/2}$



$\left(-\sqrt{3/2},\ \frac{5}{2}\right)$

$\left(\sqrt{3/2},\ \frac{5}{2}\right)$



The quantity to be minimized is length. From the diagram, you can see that x varies between 0 and 30.

Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached to a single stake, running from the ground level to the topof each post. Where should the stake be placed to use the least amount of wire?

$W = y + z$

$\sqrt{y^2} = \sqrt{144 + x^2}$ $\sqrt{z^2} = \sqrt{784 + (30-x)^2}$

$y = \sqrt{x^2+144}$ $z = \sqrt{(30-x)^2+784}$

$W = \sqrt{x^2+144} + \sqrt{(30-x)^2+784}$

$(x^2+144)^{1/2} + ((30-x)^2+784)^{1/2}$

$\frac{dW}{dx} = \left(\frac{x}{\sqrt{x^2+144}} + \frac{x-30}{\sqrt{x^2-60x+1684}} = 0\right)(\sqrt{x^2+144})(\sqrt{x^2-60x+1684})$

$x\sqrt{x^2-60x+1684} + (x-30)(\sqrt{x^2+144}) = 0$

$-(x-30)(\sqrt{x^2+144}) \to -(x-30)(\sqrt{x^2+144})$

$\left(x\sqrt{x^2-60x+1684}\right)^2 = \left((-x+30)(\sqrt{x^2+144})\right)^2$

$x^2(x^2-60x+1684) = (-x+30)^2(x^2+144)$

$x^4 - 60x^3 + 1684x^2 = x^4 - 60x^3 + 1044x^2 - 8640x + 129,600$

$640x^2 + 8640x - 129,600 = 0$

$320(x-9)(2x+45) = 0$

$x = 9$ $x = \cancel{-22.5}$

$- \; 9 \; +$ $x = 9$

Least amt of wire when the stake is 9 m from the 12 ft pole.

Find two positive numbers such that the sum of the first and twice the second is 100 and their product is a maximum.

$n = xy$ max

$x + 2y = 100$

$y = \frac{100 - x}{2} = 50 - \frac{1}{2}x$

$n = x\left(50 - \frac{1}{2}x\right)$

$n = 50x - \frac{1}{2}x^2$

$n' = 50 - x = 0$

$\boxed{x = 50}$

$+ \quad 50 \quad -$

$50 + 2y = 100$

$\boxed{y = 25}$